

1 Modeling Vehicle Interactions during Freeway Ramp Merging in  
2 Congested Weaving Section  
3  
4

5 Xia Wan, Ph.D., (Corresponding Author)  
6 Department of Civil and Environmental Engineering,  
7 University of Wisconsin-Madison, Madison,  
8 1415 Engineering Drive, USA, WI 53706,  
9 Phone: 1-608-556-4289  
10 E-mail: wan5@wisc.edu  
11

12  
13 Peter J. Jin, Ph.D., Postdoctoral Fellow,  
14 Department of Civil, Architectural, and Environmental Engineering,  
15 The University of Texas at Austin, Austin, TX 78701  
16 Phone: 1-512-232-3124  
17 Email: jjin@austin.utexas.edu  
18

19  
20 Fan Yang, Ph.D. Candidate,  
21 Department of Civil and Environmental Engineering,  
22 University of Wisconsin-Madison, Madison,  
23 WI 53706, USA  
24 E-mail: [fyang29@wisc.edu](mailto:fyang29@wisc.edu)  
25  
26

27 Bin Ran, Ph.D.  
28 Professor  
29 School of Transportation, Southeast University  
30 No.2 Si Pai Lou, Nanjing 210096, China  
31 and  
32 Civil and Environmental Engineering  
33 University of Wisconsin - Madison  
34 Madison, WI 53706, USA  
35 bran@wisc.edu  
36  
37

38 Words: 6390 Words + 5 Figures + 1 Tables = 7890 Words  
39 Submitted November 15<sup>th</sup>, 2013  
40  
41  
42  
43  
44  
45  
46

1 **ABSTRACT**

2 The difficulty to capture the interactions between vehicles in congested merging area during lane  
3 change process hinders the development of microscopic traffic modeling. The main goal of this  
4 paper is to quantify the interactions between vehicles during lane change in congested weaving  
5 section and try to incorporate them into vehicles' (merging vehicles, their putative leaders (PL)  
6 and putative followers (PF)) acceleration-deceleration models. Based on the findings by  
7 analyzing the US101 data, the yielding behavior of merging vehicles' putative leader (PL) for  
8 merging cooperation and the lateral separation between vehicles in weaving section (both  
9 features largely ignored in the past research) are introduced into our proposed acceleration-  
10 deceleration models. The visual angle information is employed as the stimuli in our models to  
11 present the effect of lateral movement of vehicles. The particular car following behaviors are  
12 incorporated in the modeling, such as the PF gradually changing its car-following leader from PL  
13 to merging vehicle, which depends on the relative locations of vehicles.

14 The model calibration and validation results based on field data have demonstrated that  
15 the proposed acceleration-deceleration models qualitatively simulated the driving behavior of  
16 vehicles in the lane change process and obtained acceptable training and testing errors. To verify  
17 the cooperation behavior of PL in lane changing, a comparison result of the proposed PL  
18 acceleration-deceleration model and a base model implied that incorporating the effects of  
19 merging vehicle on PL into the models could enhance the realism of the lane changing model.  
20 Findings from this study could contribute to the understanding of interactions between vehicles  
21 during complex lane-changing behavior.

22  
23 Key Words: Merging Process, Interactions between Vehicles, Yield Behavior, Lateral Separation,  
24 Congested Weaving Section  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45

# 1. INTRODUCTION

The lane change behavior of vehicles is complex and the fundamental part of microscopic traffic flow simulation model, which has attracted increased attention recently. A significant number of works have been done to build microscopic merging behavior related models. However, the difficulty to capture the interactions between vehicles during a complex lane change behavior hinders the development of microscopic traffic simulation (1-4).

Only a limited number of studies in the literature deal with vehicle interactions in detail. In existing studies, to handle the vehicle interactions in lane change modeling, lane changes are usually classified to three types based on the gap distance between putative leader (PL) and putative follower (PF): Free Lane Change, Cooperative Lane Change and Forced Lane Change (5-8). The key assumption of this method, the existence of interactions between vehicles mainly depends on the gap distance between PL and PF and the interactions only could trigger the PF to yield to merging vehicles, has some limitations. First, in our previous merging behavior analysis, in congested weaving section, the situation that the PL speed up to cooperate the merge process is frequently observed (9). On the other hand, interactions among the merging vehicle, its PL and its PF could not be unaffected by surrounding traffic environment. For example, the probability of yielding of PF is not only related with the locations of merging vehicle and PL, but also the vehicle closely following it, which could decrease the yielding possibility due to safety consideration. Thus, this paper aims to analyze and model the interactions between merging vehicles, their PL and PF, considering the effects of other vehicles around them.

In the real traffic world, every vehicle makes the decision and reacts based on its perceived intentions of the other vehicles. The lateral movement of vehicles in the lane change process was rarely analyzed in the past, which contains valuable information for other drivers' perception and judgment. In the merge process, the merging vehicles have to follow their PL by accompanying with lateral movement while the PF need gradually to set the merging vehicle as its new following leader based on the lateral movement of itself. The lateral effects have been proved widely existing and influential in realistic car-following behavior (10, 11). Especially in the area- a weaving section we studied, there are three types of vehicles with different route plans: going through, merging in and exiting the highway mainline. Their different route plans lead to significant lateral separation between the follower and leader driving on the same lane. Angular velocity contains the lateral information, which was used by Kou to build acceleration-deceleration model of merging vehicles (12). This paper would adopt the visual angle information as the stimulus between vehicles during the merge process.

This study first analyzes driving behavior of vehicles in longitudinal and lateral directions with observed vehicle trajectory data, and then builds the dynamic acceleration-deceleration models for merging vehicle, PL and PF by incorporating two dimensional interactions. To simplify the complex lane change condition, the study only focuses on the interactions between vehicles after the merging vehicles merge in their accepted gap. The processes of the merging vehicles driving through their rejected gap are not included in this study, because these conditions are more like overtaking executions. Second, we present the literature review about interactions analysis in lane change and acceleration-deceleration modeling. The third section is the description of the NGSIM data and empirical driving behavior analysis in longitudinal and lateral directions. Fourth, it shows the acceleration-deceleration model of merging vehicle, PL and PF incorporating interactions between vehicles. The model calibration and testing are discussed in section five. The conclusion and future work are addressed in section six.

## 1 2. LITERATURE REVIEW

### 2 2.1 Interaction between Vehicles during Merging Action

3 In the merging area, there are two interactive traffic streams: merging vehicles and  
4 mainline driving-through vehicles. In the early lane change models, the complex dynamic  
5 interactive behavior is simplified by assuming that merging traffic has no influence on the  
6 mainline traffic (13, 14). Nevertheless, many observations indicate that driving-through vehicles  
7 express a kind of cooperative behavior by changing to the inner lanes or by yielding to create  
8 gaps for merging vehicles (15-17). It is clear that merging drivers adjust their speeds according  
9 to speeds of their putative leader (PL) and putative follower (PF) on the target lane. The presence  
10 of yield behavior of mainline vehicles indicates the existence of interaction between merging  
11 vehicles and mainline traffic during the lane change process. However, only a limited number of  
12 studies in literature deal with vehicle interactions in lane change modeling. To our best  
13 knowledge, though we noticed the PL shows yield behavior to merging vehicles in the congested  
14 merging area, most of the existing research only studies the yield behavior of the mainline lag  
15 vehicle (9).

16 Researcher applied the game theory to model the interaction between the merging vehicle  
17 and its PF during the merging process. Kita et al. (2002) modeled the interaction between the  
18 merging vehicle and its PF as a two-person non-zero-sum non-cooperative game with complete  
19 information (18). The merging vehicle decides whether to move-in or pass, and the mainline PF  
20 decides whether to give way or not. However, this research only considered the situation that the  
21 vehicle driving on the mainline changes to the inner lane to give way to merging vehicle, and it  
22 assumes the two vehicles having conflict keep constant speed during the merge process. Liu  
23 (2007) used the game theoretical approach to model merging and yielding behavior at freeway's  
24 on-ramp section (19). The strategy of the competition between merging vehicle and its PF is that  
25 the PF aims to maintain their initial car-following state and minimize speed variations, and the  
26 merging vehicle wants to join mainline traffic in the minimal time possible. A bi-level estimation  
27 methodology was used to search the Nash equilibrium for the two players. The demand on too  
28 much information in the game theoretic modeling limited its usage in the microscopic simulation.

29 Other way to modeling the interactions between vehicles is to divide the merge process as  
30 three types: free, cooperative and forced lane change. Hidas (2005) established a simulation  
31 model, called ARTEMiS, to model interactions by using the autonomous agent (7). In the model,  
32 when the gap is less than the given minimum free lane change gap, the PF may act as giving way,  
33 slowing down, or not giving way, whose willing depends on the level of congestion and the  
34 individual driver's characteristics. The feasibility to slow down for the PF is calculated by the  
35 space gap between PF and the merging vehicle at the end of the deceleration period, but the  
36 effect of the vehicle behind the PF is not considered. The action choice (to yield or not to yield)  
37 of the PF is determined by the checking sequence of the model (when free lane change is  
38 impossible), since no vehicle communication is considered in the model. Ben-Akiva and  
39 Choudhury (2009) proposed a combined merging model which includes normal, courtesy and  
40 forced lane change (20). The model incorporated the courtesy deceleration of the PF if a normal  
41 lane change is impossible. The result of courtesy or forced lane change is modeled as instant  
42 deterministic choice of vehicles, ignoring the negotiations between drivers during the process.

43 Another method to realize interaction modeling is setting the interactions into the  
44 stimuli-response psychophysical concept and modifying the conventional car-following models  
45 to suit the lane change background. Sarvi (2007, 2011) built a freeway ramp merging micro-

1 simulation model, in which the acceleration-deceleration of merging vehicles and PF are linearly  
2 related with the stimuli from other vehicles (21, 22). For example, the acceleration-deceleration  
3 of PF is under the stimuli of the speed of merging vehicle, the speed of PL, distance between PF  
4 and merging vehicle and distance between PF and PL. The simulation models were calibrated  
5 with observed field lane change data.

6 Other researcher used distinct methods to model the interactions between vehicles. *Wang*  
7 (2005) presents an interaction-based model, in which the decision of PF (whether or not to  
8 provide courtesy yielding) is picked up randomly from a binomial distribution with a given  
9 probability parameter (8). Sun (2010) adopted a sequence of “hand-shaking” negotiations to  
10 handle the competition and cooperation among vehicles on arterial streets (1). Sun used the gap  
11 distance between merging vehicle and PF to determine that the PF yields or not to the merging  
12 vehicle.

13 In summary, most existing researches only model the possibility of the PF’s yielding  
14 behavior. This paper attempts to capture the interaction among the merging vehicles, PL and PF  
15 in merge process and model the courtesy yielding of PL. We choose the stimuli–response  
16 concept and car following theory to model the interactions because it can continuously apply the  
17 interactions between vehicles into the merging process modeling.

## 18 **2.2 The Effect of Lateral Movement in the Acceleration-Deceleration Model**

19 The longitudinal movement of vehicle in car following model is always a hot topic for  
20 researchers. The conventional car-following theory holds an assumption that vehicles travel in  
21 the middle of a single lane. However, the lateral separation of vehicle during the lane change  
22 process could not be ignored since the merging vehicle and its PL/PF drive on different lanes  
23 prior to the merge maneuver, which is more like the scenario under the staggered car following  
24 condition.

25 Recently, a few studies have been done focusing on the effect of lateral separation during  
26 car following. Gunay(2007) proved the existence of lateral discomfort during vehicles movement,  
27 and proposed the staggered car following theory, in which the car following movement of the  
28 following vehicle is under the impact of the off-center effects of its leader (10). Jin (2010) built a  
29 non-lane-based car following model to account the lateral separation characteristics between the  
30 leader and follower (11). The visual angle information was set as the stimulus during the car  
31 following condition.

32 Most lane change models assume the lane change execution is an instantaneous action  
33 after the gap selection, and they pay rare attention to the lateral separation of vehicles during  
34 interacting. However, the lateral movement of vehicle carries considerable information during  
35 the vehicle’s communication. Thus, in this report we attempt to introduce the effect of lateral  
36 separation into the lane change acceleration-deceleration modeling.

## 37 **3. DATA SET AND EMPIRICAL ANALYSIS**

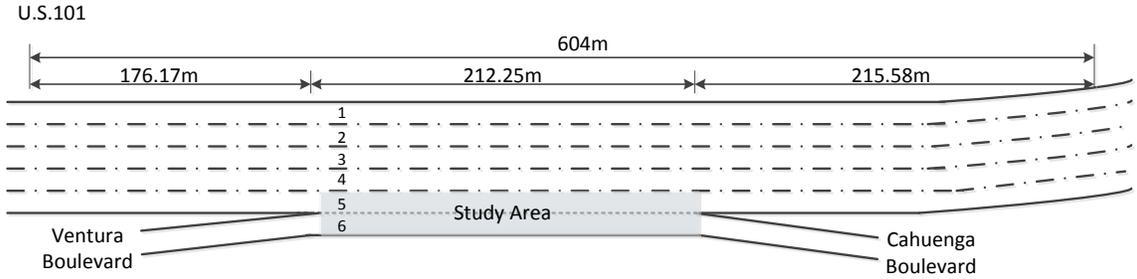
### 38 **3.1 General Description of NGSIM Data**

39 This study uses vehicles trajectory data collected on a five-lane freeway section with an on-ramp  
40 from Ventura Boulevard and an off-ramp to Cahuenga Boulevard on U.S. Highway 101  
41 (Hollywood Freeway), Log Angeles, California, USA (see figure 1a for the geometric layout). It  
42 is a part of FHWA’s Next Generation Simulation (NGSIM) program. The total length of the

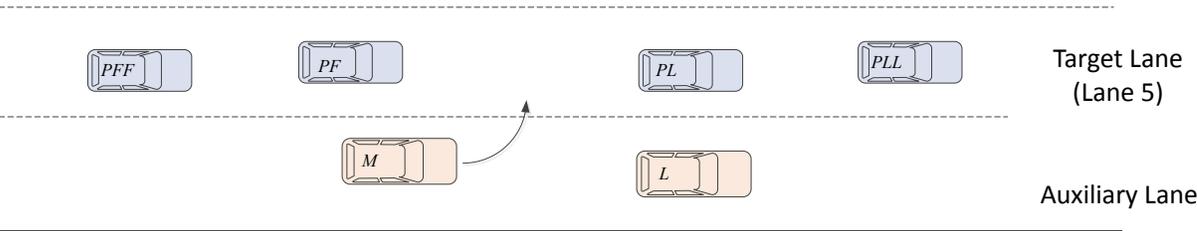
1 observation area is 604 m, and the vehicle trajectories are updated in every 0.1 second from 7:50  
 2 to 8:35 a.m. on June 15, 2005. In this study, we focus on the weaving section, whose length is  
 3 212.25 m. In the 45 minutes observation time, the weaving section is in transition (7:50-8:05 am)  
 4 and congestion (8:05-8:35 am) (23). The speeds of mainline traffic vary from 27.3 to about 51.50  
 5 km/h (average value is around 41.03 km/h) during the 45 minutes; while the average speed of the  
 6 on-ramp merging vehicles when arriving the auxiliary lane is around 49.48 km/h. Thus, this data  
 7 set could be identified as congested weaving section.

8 The NGSIM data set distinguishes three vehicle classes: motorcycles, cars, and trucks.  
 9 Due to the low percentage of quantities of motorcycles and trucks in this data set (the total of  
 10 them is less than 3 percent), they were excluded in this study. 398 merging vehicles were  
 11 extracted out when a total of 11,779 vehicles were processed. In figure 1b, we sketched the study  
 12 area and vehicle layout, where the merging vehicle (M) interacts with its putative leader (PL) and  
 13 putative follower (PF) on the target lane and its leading vehicle (L) on the auxiliary lane. The  
 14 leader of PL and the follower of PF also are investigated in this study, which are called PLL and  
 15 PFF, respectively. In a weaving section, there are three types of vehicles having different route  
 16 plans: driving-through vehicles, merging vehicles and exiting vehicles.

17 Merging vehicles' merge gap selection and merge tactics are influenced by the traffic  
 18 condition on the target lane. At the left of this section, we would analyze the effect of merging  
 19 vehicle on mainline traffic under the condition of vehicle interactions.  
 20



(a) Data site-U.S.101



(b) Vehicle layout and notations

FIGURE 1 Data collection site (a) and the related vehicles (b).

**3.2 Longitudinal Interactions during Merging Process**

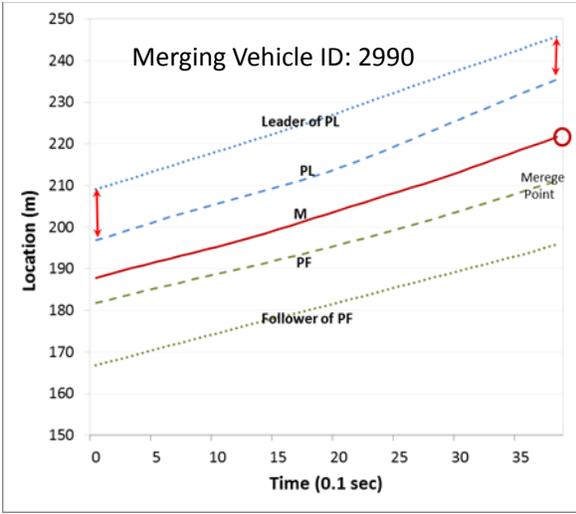
Previous lane change studies pointed out, when the gap between the PL and PF is smaller than minimum required gap for free lane change, the cooperative lane change condition is activated along with the yielding action of the PF. In this section, we explore the yielding behavior of

1 mainline traffic with field data from the congested area.

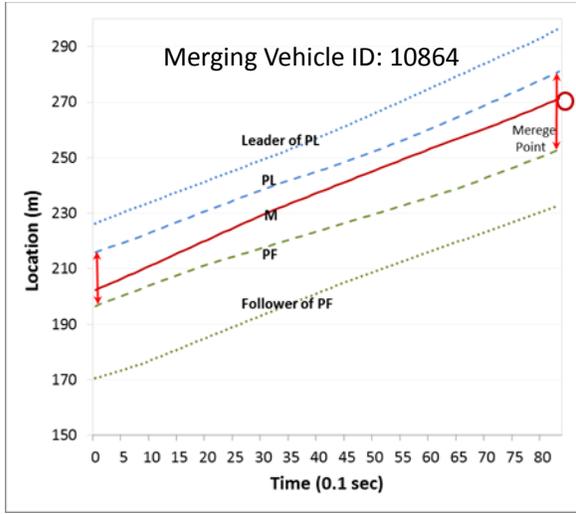
2 First, the proportion of two types yielding behavior of mainline traffic (slowing down or  
3 changing to the inner lane to create a gap for merging vehicle) is examined. Almost 3000  
4 driving-through vehicles which once drive on the lane five (in figure 1a) are collected by  
5 screening based on their travel trajectory. The results show that there are only 51 of them  
6 changing to the inner lane in the 604 m observation area, and only 9 of these 51 could be  
7 considered as the yielding-based lane change vehicles, which were involved to be PFs or PLs in  
8 the study area. The possible reason for the low rate of yielding-based lane change in the  
9 observation area is that the anticipated yielding lane changes commonly occur prior to the  
10 effective ramp area, and the remaining driving-through vehicles either have to travel or  
11 intentionally persist in traveling on the merging vehicle target lane. Here, only the yielding  
12 behavior of mainline traffic vehicles by slowing down will be in detail analyzed and modeled,  
13 since the lane change-related yielding behavior rarely executes (manifested by its low  
14 percentage).

15 Merging vehicle getting into the gap between PL and PF would disturb the car-following  
16 status of them. Before the lane change maneuver, the PL needs to estimate the distance from the  
17 merging vehicle to itself to avoid collision for the anticipated merge, while the PF has to be  
18 ready for the anticipated merge action when it notices the strong merge intention of merging  
19 vehicle. During the merging, the merging vehicle switches to the PF's new leader. We have  
20 noticed not only the PF is impacted by pressure of anticipated merge requirement, but also the  
21 PL shows cooperative yielding for merge vehicles. Fig. 2 shows two random examples of the  
22 merging process including yielding behavior of mainline traffic, during which the PL (PL of  
23 merging vehicle ID 2990) or PF (PF of merging vehicle ID 10864) obviously yields to merging  
24 vehicle, respectively. "0" at x axis in Figure 2 indicates the time point when merging vehicle gets  
25 into its accepted gap, and the end point of the time axis is the time point when the geometrical  
26 center point of the merging vehicle crosses the lane line shared by the auxiliary lane and target  
27 lane. The yielding behavior of the mainline traffic is manifested by dramatically acceleration  
28 (PL) or deceleration (PF) after the presence of merging vehicle, and the increase or decrease of  
29 the distance gaps between among them.

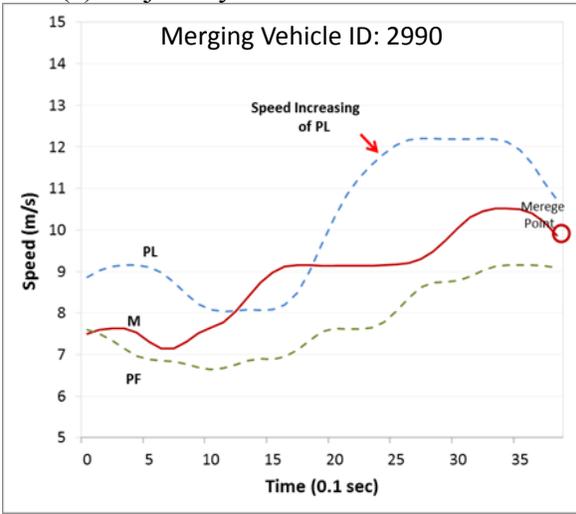
30 We use the change rate of gap distance between PLL and PL during merge process to  
31 quantify the yield behavior of PL. Excluding the PLL's obvious deceleration case, in 67 merge  
32 processes, the gap distance between PLL and PL reduces by more than 15% percent due to the  
33 acceleration of PL for merge cooperation. Similarly, for the PF, in 86 merge processes the gap  
34 distance between PL and PF increases by more than 15% percent due to the deceleration of PF  
35 after removing the effect of the acceleration of PL. Considering that the total samples observed  
36 are 398, both of the yielding behavior of PL and PF should be included in the interaction  
37 modeling.



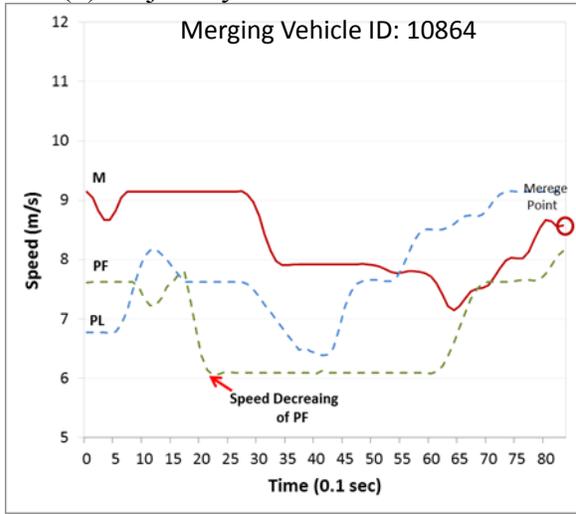
(a) Trajectory of interactive vehicles



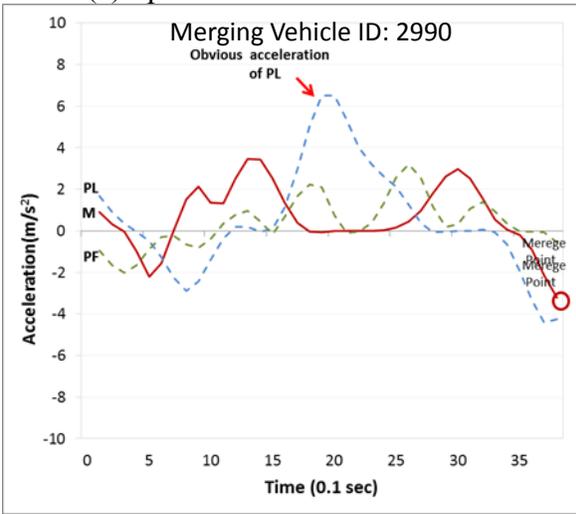
(b) Trajectory of interactive vehicles



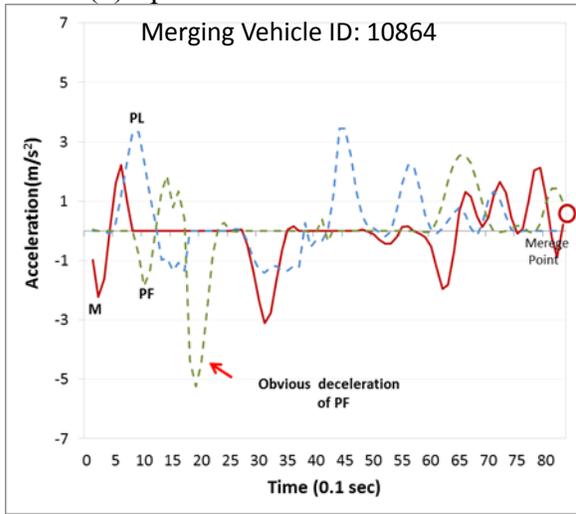
(c) Speed of interactive vehicles



(d) Speed of interactive vehicles



(e) Acceleration of interactive vehicles



(f) Acceleration of interactive vehicles

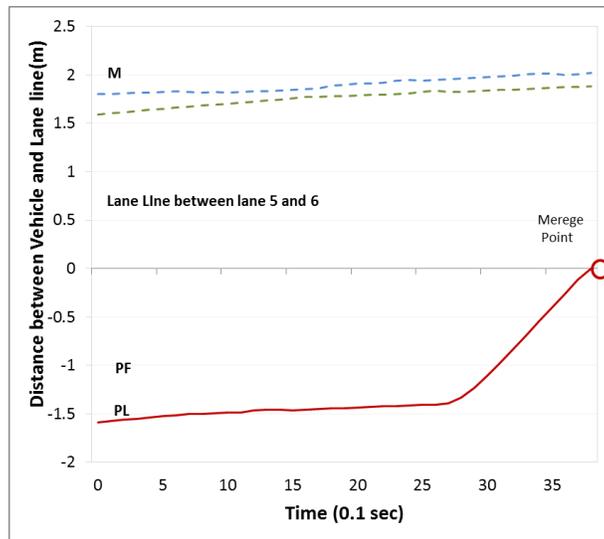
1  
2

FIGURE 2 PL (merging vehicle ID 2990) and PF (merging vehicle ID 10864) Courtesy  
Yielding example.

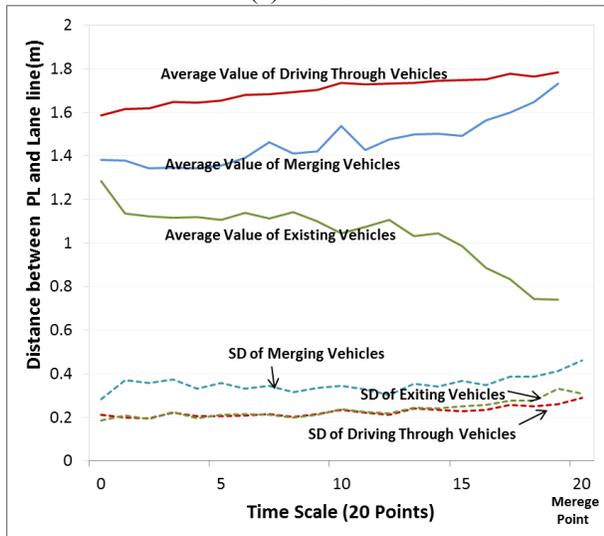
1 **3.3 Lateral Interactions during Merging Process**

2 In the merge process, when the PL and PF notice the existence of merging vehicle, they have the  
 3 tendency to shift further away from the right side lane line to avoid the possible collision.  
 4 Though the analysis results in section 3.2 show that only the PL of merging vehicle (ID 2990) is  
 5 significantly affected by merging vehicle to perform yielding, figure 3a shows both the PL and  
 6 PF shift further away from the right side lane line during the lane change process. The evidence  
 7 indicates both the PL and PF are under the influence of the merging vehicle.

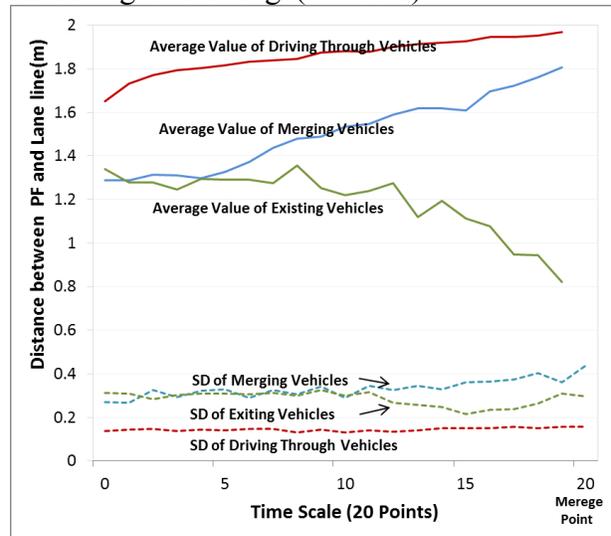
8 The U.S.101 data was further examined to reveal whether the lateral movement of PL and  
 9 PF is a common phenomenon during merge process. We rescale the total time (between the time  
 10 point when the merging vehicles start to involve in their accepted gap and the time point when  
 11 they right merge into their target lane) and divide it to 20 sections of same length. Figure 3b and  
 12 Figure 3c show the lateral movement of PL and PF for three different types  
 13



(a) Lateral movement of vehicles during lane change (ID 2990)



(b) Average lateral movement of PL for different vehicles types



(c) Average lateral movement of PF for different vehicles types

FIGURE 3 Lateral Interaction during the merging process.

1 of vehicles separately in such a weaving section. The figures show that the driving-through  
2 vehicles tend to keep away from the merging vehicle no matter what role they play (either as PL  
3 or PF), and their lateral movement are more noticeable right after they sense the presence of the  
4 merging vehicle. Meanwhile, the merged vehicle and exiting vehicle conduct lateral movement  
5 by coordinating with their route plan. Even the vehicles all travel on the target lane, there could  
6 be significant lateral separation between the leader and follower as the figure 3b and figure 3c  
7 show. After comparing the different lateral movement trajectories of PL and PF, we could tell  
8 that both the merged and exiting vehicles are closer to their own lateral destinations, respectively,  
9 at the same time point of the 20 point-scale dimension when they function as PL. This trend is  
10 reasonable since the PL travels longer than the PF across a merge event, which gives  
11 merged/exiting vehicles more time to move toward their target lateral destination.

12 The lateral movement analysis results illustrate that the effect of merging vehicle on its  
13 PL and PF exist and is manifested by the lateral movement of them during the lane change  
14 process. Another notable finding is that the lateral separation occurs frequently during the merge  
15 process, which should be included in the interaction modeling.

## 16 4. MODELING

17 In this study, the interactions between vehicles are considered as continuous actions across the  
18 whole merge process. Because of the close relationship between lane change and car-following  
19 in the merging area, it is necessary to incorporate both of them in the merging process micro-  
20 simulation model. The merging vehicle getting onto the target lane leads to a distinct space  
21 reduction between its PL and PF and disturbance to current car-following condition. Before the  
22 merging vehicle arrives it accepted gap, its PL and PF maintain a car-following status under its  
23 own route plan. Immediately upon the emergence of the merging vehicle, the entire merging  
24 vehicle, PL and PF have to adjust current car-following condition and decide a set of moves to  
25 avoid potential collision for the following merge maneuver. The action strategies for vehicles  
26 during merge are assumed as follows:

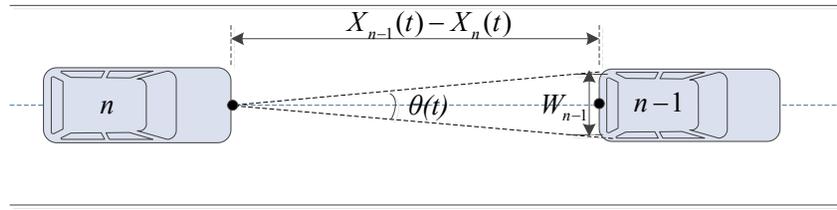
- 27 • The merging vehicle (M): it sets its PL as its following vehicle for lane change  
28 preparation, and keeps a safe distance to its PF and its leading vehicle (L) on current lane;
- 29 • The PL: it follows its leading vehicle (PLL) and keeps a safe distance from the merging  
30 vehicle (M) for safety issue;
- 31 • The PF: it gradually changes its following vehicles from PL to M, and keeps a safe  
32 distance to the vehicle following it (PFF).

33 During a merge process in a weaving section, the merging vehicle, PL and PF do not  
34 travel on the middle of the same lane, thus the lateral separation between vehicles makes the  
35 conventional car-following model inapplicable in merge condition. Also, the PL and PF drivers  
36 also pay more attention to the merging vehicle's lateral movement and use it to predict the  
37 movement of merging vehicle. New acceleration-deceleration models are built in this section  
38 based on visual angle information to simulate the acceleration-deceleration of the merging  
39 vehicle, PL and PF during the merging process, which considers the longitudinal space distance,  
40 lateral separation, speed difference and effect of different vehicles types.

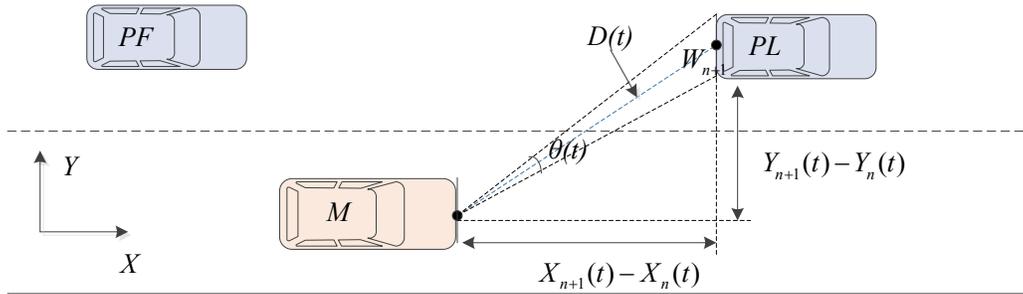
### 41 4.1 Car Following with Visual Angle Information and Notations

42 During the lane change process, the main task of vehicles is to avoid collision with other vehicles  
43 and follow their route plan based on car-following. Many researchers reported that the dominant

1 perceptual factor (the stimulus) is the rate of change of visual angle in the car-following situation  
 2 (11, 12, 24). The change rate of visual angle, called angular velocity, could be expressed as  $\frac{d\theta(t)}{dt}$   
 3 in Figure 4.



4 (a) Basic car-following behavior  
 5  
 6



7 (b) Staggered car-following behavior  
 8

9 FIGURE 4 Visual Angle Information during Merge Process  
 10

11 When the leader and follower vehicles travel on the middle of the same lane (shown in  
 12 Fig. 4a), the equation to calculate the visual angle  $\theta(t)$  and angular velocity are derived:

13 
$$\theta(t) \approx \frac{W_{n-1}}{|X_{n-1}(t) - X_n(t)|} \quad (1)$$

14 
$$\frac{d\theta(t)}{dt} \approx \frac{W_{n-1}}{(X_{n-1}(t) - X_n(t))^2} \cdot [V_{n-1}(t) - V_n(t)] \quad (2)$$

15 Where,  $W$  is the width of vehicle  $n-1$ ;  $X_{n-1}(t)$  and  $X_n(t)$  are the longitudinal location of  
 16 vehicle  $n-1$  and vehicle  $n$ , respectively, at time  $t$ ;  $V_{n-1}(t)$  and  $V_n(t)$  are the speed of vehicle  $n-1$   
 17 and vehicle  $n$ , respectively, at time  $t$ . The detailed derivation could be found in reference (19).

18 Considering the lateral separation during the lane change process, the visual angle and  
 19 angular velocity shown in Figure 4b, with merging vehicle (M) and PL as an example, are  
 20 modified as:

21 
$$\theta(t) \approx \frac{W_{PL}}{D(t)} \quad (3)$$

22 
$$\theta'(t) \approx \frac{d\theta(t)}{dt} = \frac{W_{PL}}{D(t)^2} \cdot [V_{PL}(t) - V_M(t)] \quad (4)$$

23 Where  $D(t)$  is the distance between PL and M at time  $t$ , where  $D(t)$  is calculated as:

24 
$$D(t) = \sqrt{(X_{PL}(t) - X_M(t))^2 + (Y_{PL}(t) - Y_M(t))^2} \quad (5)$$

25 Where,  $Y_{PL}(t)$  and  $Y_M(t)$  are the lateral locations of PL and M, respectively, at time  $t$ .

26 The fundamental psychophysical concept of car-following models is appropriate to model

1 acceleration-deceleration behavior with stimulus. The basic car following model is written as:

$$2 \quad a_n(t+T) = \lambda \cdot \theta'(t) \quad (6)$$

3 Where  $a_n(t+T)$  is the acceleration of vehicle  $n$  at time  $t+T$ ;  $T$  is the reaction time of  
4 driver.

## 5 **4.2 Acceleration-Deceleration Modeling Incorporating Interactions between Vehicles**

6 The driving behaviors of vehicles traveling in the weaving section are under the impact of the  
7 interactions between vehicles in the congested condition. The dynamic acceleration-deceleration  
8 of merging vehicle, PL and PF in the longitudinal direction are modeled in this section. The  
9 vehicles type information is contained in their lateral movement, serving as the input of these  
10 models.

### 11 *4.2.1 Dynamic acceleration-deceleration model of merging vehicle*

12 The acceleration and deceleration behavior of merging vehicles involves two tasks, including  
13 following the PL and keeping a safe distance from other vehicles around it to get appropriate  
14 space for lane change execution. Here, the PF and L (the leading vehicle on auxiliary lane) are  
15 set as the required safety space constraints for merging vehicle. The basic car-following model  
16 shown in equation 6 is linearly expanded to acceleration-deceleration model of merging vehicle,  
17 incorporating the car following (first term) and the influence of the PF (second term) and L (third  
18 term):

$$19 \quad a_M(t+T) = \lambda_1 \theta'_{PL}(t) - \lambda_2 \cdot \min[0, \theta_{DesPF} - \theta_{PF}(t)] + \lambda_3 \cdot \min[0, \theta_{DesL} - \theta_L(t)] \quad (7)$$

20 Where  $\theta'_{PL}$  is the visual angular velocity of PL from the merging vehicle's view,  
21 calculated with equation 4;  $\theta_{PF}(t)$  and  $\theta_L(t)$  are the visual angle of PF and L, respectively, from  
22 the merging vehicle's view at time  $t$ , calculated with equation 3;  $\theta_{DesPF}$  and  $\theta_{DesL}$  are the  
23 desired visual angle from merging vehicle to its PF and L, respectively, which are constant value  
24 needed to be calibrated in this model;  $\lambda_1$ ,  $\lambda_2$  and  $\lambda_3$  are used to determine the weights of these  
25 three stimuli.

26 The first term of equation 7 represents the merging vehicle following its PL during the  
27 lane change process. The visual angular velocity  $\theta'_{PL}$  contains speed difference, longitudinal and  
28 lateral location information of M and its PL. The second term describes the response of merging  
29 vehicle to the close PF behind it, and its effect could only be acceleration. When the visual angle  
30  $\theta_{PF}(t)$  is less than the desired visual angle  $\theta_{DesPF}$ , it motivates the merging vehicle to speed up  
31 for the safety issue. The lateral separation information is contained in the visual angle. The third  
32 term represents the response of merging vehicle to its leading vehicle driving (L) on the auxiliary  
33 lane, which could only be deceleration stimuli for keeping safety space. The concept is the same  
34 as the second term, so here we skip the redundant description.

### 35 *4.2.2 Dynamic acceleration-deceleration model of Putative Follower (PF)*

36 During the merge process, the merging vehicle's PF should change its following leader from PL  
37 to merging vehicle gradually. This process depends on the variation of relative lateral and  
38 longitudinal positions of vehicles.

39 In the lateral direction, when the merging vehicle is laterally close enough to PF, the PF  
40 would set merging vehicle as its new leader because of its strong merge intention. To capture this  
41 lateral distance effect, the lateral related position factor  $\frac{\alpha \cdot l}{|Y_M(t) - Y_{PF}(t)|}$  is employed for the PF to  
42 determine its following leader. Here,  $l$  is the average lane width of auxiliary lane and target lane,

1 and  $\alpha$  is a parameter needed to estimate with field data. When  $\frac{\alpha \cdot l}{|Y_M(t) - Y_{PF}(t)|}$  is large enough, the  
 2 merging vehicle is the only leader of PL. Otherwise, the PF is under the stimulus coming from  
 3 both the merging vehicle and PL.

4 In the longitudinal direction, if the merging vehicle is extremely close to the PF, it would  
 5 results an unrealistically significant deceleration for PF in a conventional car-following model.  
 6 Actually, the situation that the space between merging vehicle and it's PF is extraordinarily tiny  
 7 is common in the congested area when the merging vehicle meets a rejected gap prior to  
 8 merging. To overcome this problem in car-following theory under lane change situation, a  
 9 relative longitudinal location factor  $\frac{\beta \cdot |X_M(t) - X_{PF}(t)|}{|X_{PL}(t) - X_{PF}(t)|}$  is adopted here to present a resistance of PF  
 10 to the unrealistic deceleration. This factor means, when plenty of space is left between merging  
 11 and PL, instead of applying significant deceleration, the PF trusts the nearby merging vehicle  
 12 would adjust its relative position prior to the lane change execution.  $\beta$  is a parameter for  
 13 calibration. On the other hand, the distance between PF and its follower (PFF) is incorporated in  
 14 the model, which plays a crucial role for PF to check whether a yielding action is feasible or not.  
 15 Overall, the dynamic acceleration-deceleration model of PF, expressed by equation 8 and 9, is  
 16 obtained.

17 If  $\frac{\alpha \cdot l}{\Delta Y_{M-PF}(t)} \geq 1$ ,

$$18 \quad a_{PF}(t+T) = \lambda_4 \cdot \theta'_M(t) - \lambda_5 \cdot \min[0, \theta_{DesPFF} - \theta_{PFF}(t)] \quad (8)$$

19 If  $\frac{\alpha \cdot l}{\Delta Y_{M-PF}(t)} < 1$ ,

$$20 \quad a_{PF}(t+T) = \lambda_6 \cdot \min[1, \frac{\beta |X_M(t) - X_{PF}(t)|}{|X_{PL}(t) - X_{PF}(t)|}] \cdot \theta'_M(t) + \lambda_7 \cdot \left\{ 1 - \min[1, \frac{\beta |X_M(t) - X_{PF}(t)|}{|X_{PL}(t) - X_{PF}(t)|}] \right\} \theta'_{PL}(t) \\ 21 \quad - \lambda_8 \cdot \min[0, \theta_{DesPFF} - \theta_{PFF}(t)] \quad (9)$$

22 Where,  $\theta'_M(t)$  and  $\theta'_{PL}(t)$  are the visual angular velocity of Merging vehicle and PL,  
 23 respectively, from the PF's view at time t;  $\theta_{PFF}(t)$  is the visual angle of PFF from the PF's view  
 24 at time t;  $\theta_{DesPFF}$  is the desired visual angle from PF to its PFF, a constant value;  $\lambda_4 - \lambda_8$  are  
 25 parameters to determine the weights of stimuli in the models.

26 In equation 8, merging vehicle is the only leader of PF. The first term is for the car-  
 27 following, and the second term is for the response of close PFF. In equation 9, PF follows both  
 28 the PL and merging vehicle. The first term and the second term are for the car-following stimuli  
 29 with relative longitudinal location resistance factor, and the third term is the response of close  
 30 PFF behind PF.

### 31 4.2.3 acceleration-deceleration model of Putative Leader (PL)

32 The putative leader of merging vehicle follows its leader (PLL) in the merge process. When  
 33 merging vehicle is close enough to PL, it would encourage the PL speeds up for cooperation to  
 34 avoid possible collision in following lane changing. However, the probability of PL yielding  
 35 action is affected by the relative longitudinal and lateral position of vehicles. In the lateral  
 36 direction, the lateral separation between merging vehicle and PL could reduce the pressure  
 37 coming from the merge process, which is contained in the visual angle information. In the  
 38 longitudinal direction, the yielding possibility is lowered by the low relative longitudinal location

1 factor  $\frac{\gamma \cdot |X_{PL}(t) - X_M(t)|}{|X_{PL}(t) - X_{PF}(t)|}$ , which reflects the longitudinal location adjustment ability of merging  
 2 vehicle weighted by PL.  $\gamma$  is a calibrated constant value. All the mentioned effects are expressed  
 3 in equation 10:

$$4 \quad a_{PL}(t+T) = \lambda_9 \cdot \theta'_{PLL}(t) - \lambda_{10} \cdot \min[0, \theta_{DesM} - \theta_M(t)] \cdot \min\left[1, \frac{\gamma |X_{PL}(t) - X_M(t)|}{|X_{PL}(t) - X_{PF}(t)|}\right] \quad (10)$$

5 where,  $\theta'_{PLL}(t)$  is the visual angular velocity of PLL from the PL's view;  $\theta_M(t)$  is the visual  
 6 angle of merging vehicle from the PL's view at time  $t$ ;  $\theta_{DesM}$  is the desired visual angle from  
 7 merging vehicle to PL, a constant value obtained from observation data. The first term expresses  
 8 the PF follows its leader PLL and the second term represent the effects from the merging vehicle  
 9 modified with the longitudinal effect factor.

## 10 5. MODEL TESTING AND RESULTS

### 11 5.1 Modeling Training and Testing Method

12 In the previous section, three dynamic acceleration-deceleration models of merging vehicle, PL  
 13 and PF were proposed. In this section, these three models are calibrated and validated by  
 14 U.S.101 observation data with a genetic algorithm in Matlab. The US101 data samples (398  
 15 valid merging processes) were randomly divided to two parts with equal numbers (199 samples  
 16 each) for the model training and model testing separately. The time step in these models is set as  
 17 0.1 s according to the time step in NGSIM data.  $T$  is the reaction time of drivers, and its value is  
 18 increased from 0.5 s to 1.0 s with 0.1 s per running to search for the optimal results. The reaction  
 19 time is usually set as 1.0 s in the conventional car following model, but in lane change model it  
 20 may be shorter by considering that all the movements of vehicles are under drivers' anticipation.  
 21 In the testing step, the observed stimuli at time  $t$  are used as the input of models to get the  
 22 predicted acceleration of testing samples at time  $t+T$ . The three acceleration and deceleration  
 23 model are tested separately. When we conduct the training and testing of the merging vehicle  
 24 acceleration-deceleration model, the observed characteristics of its PL and PF are used as the  
 25 input of this model. The same process is applied for the testing of other two models.

26 Error test parameters used in this study are the mean error (ME), mean absolute error  
 27 (MAE) and Theil's inequality coefficient (U):

$$28 \quad ME = \frac{1}{n} \sum_{i=1}^n Y_i^{sim} - Y_i^{obs} \quad (11)$$

$$29 \quad MAE = \frac{1}{n} \sum_{i=1}^n |Y_i^{obs} - Y_i^{sim}| \quad (12)$$

$$30 \quad U = \frac{\sqrt{\frac{1}{N} \sum_i (Y_i^{obs} - Y_i^{sim})^2}}{\sqrt{\frac{1}{N} \sum_i (Y_i^{obs})^2} + \sqrt{\frac{1}{N} \sum_i (Y_i^{sim})^2}} \quad (13)$$

31 Where,  $Y_i^{obs}$  and  $Y_i^{sim}$  are the observed and simulated acceleration of  $i$ th subject vehicle,  
 32 respectively, with  $i$  ranging from 1 to  $N$ .

1 **5.2 Testing Results**

2 The results of proposed acceleration-deceleration models are listed in Table 1 with different  
 3 settings of driver’s reaction time.

4  
 5

Table 1 Errors of Models Calibration and Validation

Reaction Time(s)	Errors (m/s <sup>2</sup> )	Merging Vehicle(M)	Putative Follower(PF)	Putative Leader(PL)
0.5	U-Train	0.82	0.72	0.81
	ME-Train	1.14	0.40	0.56
	ME-Test	1.15	0.37	0.54
	MAE-Train	1.61	1.01	1.18
	MAE-Test	1.58	1.05	1.16
0.6	U-Train	0.78	0.72	0.76
	ME-Train	0.89	0.36	0.52
	ME-Test	0.87	0.39	0.56
	MAE-Train	1.41	1.02	1.14
	MAE-Test	1.39	1.02	1.17
0.7	U-Train	0.77	0.68	0.73
	ME-Train	0.59	0.14	0.31
	ME-Test	0.62	0.12	0.31
	MAE-Train	1.20	0.90	1.00
	MAE-Test	1.18	0.87	1.04
0.8	U-Train	0.79	0.71	0.82
	ME-Train	0.90	0.40	0.32
	ME-Test	0.94	0.42	0.31
	MAE-Train	1.39	1.02	1.04
	MAE-Test	1.40	1.02	1.05
0.9	U-Train	0.80	0.75	0.81
	ME-Train	0.96	0.14	0.36
	ME-Test	0.94	0.15	0.32
	MAE-Train	1.39	0.91	1.06
	MAE-Test	1.41	0.90	1.06
1.0	U-Train	0.81	0.74	0.86
	ME-Train	1.14	0.45	0.25
	ME-Test	1.15	0.44	0.19
	MAE-Train	1.57	1.05	1.01
	MAE-Test	1.55	1.01	1.00

6 \*199 merging process samples for calibration of three models, 199 merging process samples for test of  
 7 three models.

8 Based the simulation results shown in Table 1, reaction time  $T$  should be set as 0.7 s for  
 9 the sample set, which leads to smaller error comparing to other settings. The optimal reaction  
 10 time is consistent with the result in reference 20 (0.667 s). The estimated results for the dynamic  
 11 acceleration-deceleration models are expressed as follows:

12  
 13

For the merging vehicles:

$$a_M(t+0.7) = 2.415\theta'_{PL}(t) - 4.570 \cdot \min[0, 0.065 - \theta_{PF}(t)] + 0.174 \cdot \min[0, 0.293 - \theta_L(t)] \quad (14)$$

For the putative followers (PF):

$$\text{If } \frac{0.963 \cdot l}{\Delta Y_{M-PF}(t)} \geq 1,$$

$$a_{PF}(t+0.7) = 4.460\theta'_M(t) - 2.485 \min[0, 0.131 - \theta_{PFF}(t)] \quad (15)$$

$$\text{If } \frac{0.963 \cdot l}{\Delta Y_{M-PF}(t)} < 1,$$

$$a_{PF}(t+0.7) = 2.474 \min\left[1, \frac{0.789 |X_M(t) - X_{PF}(t)|}{|X_{PL}(t) - X_{PF}(t)|}\right] \cdot \theta'_M(t) \\ + 2.873 \left\{ 1 - \min\left[1, \frac{0.789 |X_M(t) - X_{PF}(t)|}{|X_{PL}(t) - X_{PF}(t)|}\right] \right\} \theta'_{PL}(t) \\ - 2.661 \cdot \min[0, 0.131 - \theta_{PFF}(t)] \quad (16)$$

For the putative leaders (PL):

$$a_{PL}(t+0.7) = 2.677\theta'_{PLL}(t) - 5.972 \min[0, 0.074 - \theta_M(t)] \cdot \min\left[1, \frac{1.994 |X_{PL}(t) - X_M(t)|}{|X_{PL}(t) - X_{PF}(t)|}\right] \quad (17)$$

### 5.3 Discussion

The average absolute acceleration speeds of merging vehicle, PF and PL in our modeling time interval are 1.134 m/s<sup>2</sup>, 0.840 m/s<sup>2</sup> and 0.947 ms<sup>2</sup> in the sample set, respectively. Based on the error evaluation results shown in Table 1, we noticed the merging vehicles acceleration-deceleration model holds the highest error (MAE-Test = 1.18 m/s<sup>2</sup>, U-Test = 0.77) comparing to other two models. The possible explanation is that merging vehicles bear the most complex workloads, maintaining route, **keeping safe distance between surrounding vehicles**, preparing for lane changing, which increases the difficulty to predict their reaction. The PF acceleration-deceleration model has the best performance with the lowest error value (MAE-Test = 0.87 ms<sup>2</sup>, U-Test = 0.68). It may result from that the PF could rationally react to the effects of other vehicles with more accurate information obtained by looking ahead, which makes the simulation more realistic by rational equations in physics. Overall, the omitting of heterogeneity of drivers may contribute to the errors of the proposed models. Adding the attributes of drivers into the model frame will be considered in our future study.

For the merging vehicle acceleration-deceleration model, we could tell that the merging vehicles are more sensitive to the distance between itself and its PF comparing to the one between itself and its L, as  $\lambda_2$  (4.570) is much larger than  $\lambda_3$  (0.174). The calibrated desired visual angle for PF (0.065) is smaller than that for L (0.293), which means the merging vehicles need larger space between its putative leader and itself, and they could endure smaller space gap between its leading vehicle on current lane and itself. **The conclusion from the calibrated model is reasonable. Because that the merging vehicles is particularly sensitive to their relative location to their PF for the anticipated merge maneuver and driving on main lane. However, the safety issue coming from the leader in current lane is an intermediate constraint, which will be terminated with successful merge maneuver.**

For the PF acceleration-deceleration model, based the calibration results, when the lateral distance between the merging vehicle and PF is larger than 3.23 m (0.963 times the length of the

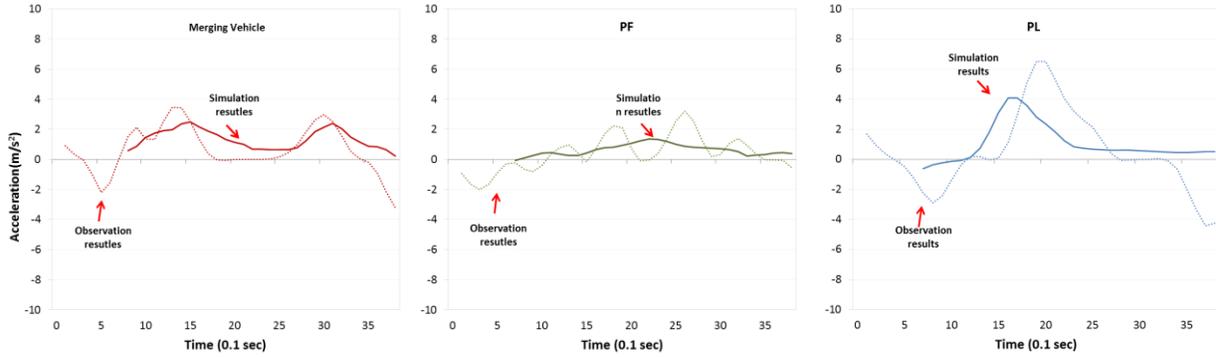
1 lane width), the PF would follow both merging vehicle and PL. Otherwise, the merging vehicle  
 2 would not sense the effect of the PL. It makes sense that when the merging vehicle become  
 3 closer to the mainline (considering the shift away of PF), the PF could feel the strong merge  
 4 intention of it and set it as potential lead. The desired visual angle from PF to its follower (PFF)  
 5 is 0.131, which is higher than the required value from merging vehicles to PF (0.065). This  
 6 means the required space distance from PF to its follower is smaller, which accord with the real  
 7 condition that lane changing vehicles need more space for safety consideration. The calibration  
 8 result of  $\beta$  is a small value (0.789) as we expected, which indicates the PF holds resistance on  
 9 dramatic deceleration as it approaches the merging vehicle due to  $\frac{0.798 \cdot |X_M(t) - X_{PF}(t)|}{|X_{PL}(t) - X_{PF}(t)|} \ll 1$ .

10 For the PL acceleration-deceleration model, the PL has intensive reaction to the merging  
 11 vehicle with a high coefficient (5.972) and low desired visual angle (0.074), which indicates the  
 12 influence of merging on the PL exists. The calibration result of  $\gamma$  is 1.994, which means the  
 13 acceleration resistance of PL to the nearby merge vehicle would disappear when the merging  
 14 vehicle almost locate in the middle of PL and PF along the longitudinal direction. To evaluate  
 15 the effect of merging vehicle on the PL, a basic acceleration-deceleration model of PL is built  
 16 without stimuli coming from the merging vehicle, written as:

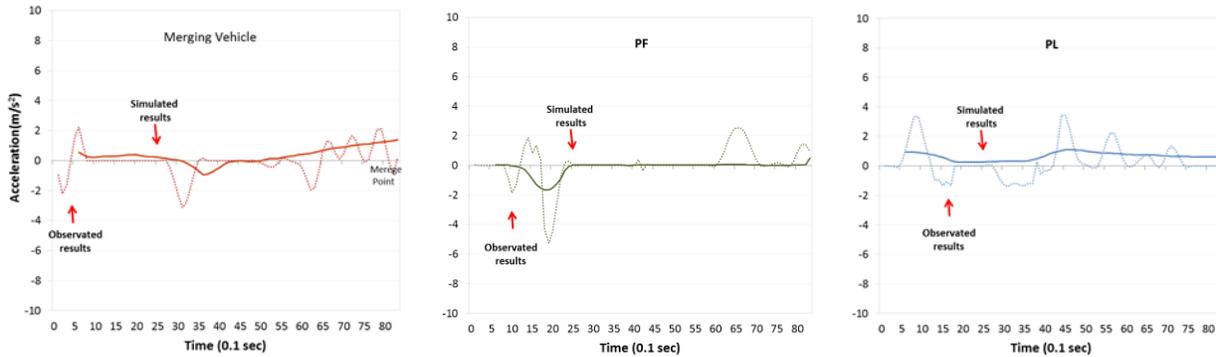
$$17 \quad a_{PL}(t+T) = \lambda_9 \cdot \theta'_{PLL}(t) \quad (18)$$

18 After training and testing, the following error test results were obtained for the base  
 19 model: MAE-Train = 1.27 m/s<sup>2</sup>, MAE-Test = 1.28 m/s<sup>2</sup> and U = 0.78. Referring to Table 1, it  
 20 could be concluded that introducing the impact of merging vehicle to the PL acceleration-  
 21 deceleration could improve the simulation accuracy by 19.5%. The testing and comparison  
 22 results are in accordance with the observation results in section three.

23 Figure 5 shows the simulated acceleration of sample vehicles, which were also showed in  
 24 Figure 2 as examples. From the comparison of the simulated results and the observed results, it  
 25 tells that the proposed model has the capability to reflect the interactions among vehicles in the  
 26 merge process. When the merging vehicle approach its PL or PF, the PL/PF responds with  
 27 yielding action whenever it is needed in the congested area. However, some further  
 28 improvements are required in our models for higher accuracy by introducing the characteristic  
 29 and random selection behavior of different drivers.



(a) Merge vehicles ID 2990



(a) Merge vehicles ID 10864

Figure 5 The Simulated Acceleration of Sample Vehicles

## 6. CONCLUSION

Although interactions between drivers are essential for modeling lane changes, they have not been incorporated explicitly into existing microscopic traffic flow model. The most significant contribution of the present study is the introduction of the yield behavior of PL and the lateral separation between vehicles (both features were largely ignored in the past) into lane change models to help capture the interactions between vehicles.

The study findings indicate some considerable conclusions in lane change process. First, obvious yield behavior of PL and PF exist in congested merge area, while in the past only the yield behavior of PF has been incorporated into the lane change modeling. Thus, the stimuli coming from the front and behind of the subject vehicles were both implemented in the proposed acceleration-deceleration models. Second, lateral separation between vehicles can not be ignored during the lane change process especially in a merge section. The proposed acceleration-deceleration models used visual angle information to extend stimuli into two-dimensional space, which can practically explain complex driving behavior with lateral movement. Third, merging vehicle, PL and PF all could clearly see the relative locations of themselves and the merging vehicle could do plenty of longitudinal location adjustment before lane change maneuver, which is significantly different from the conventional car-following models (vehicles drive in the center of one lane) and influence the yielding decision of PL and PF. To capture the particular car-

1 following behavior in lane change process, the relative longitudinal location factors are  
2 introduced in our models.

3 Finally, the model calibration and validation results based on the U.S.101 data  
4 demonstrates that the proposed acceleration-deceleration models could qualitatively predict the  
5 driving behavior of vehicles in the lane change process and obtain acceptable training and testing  
6 errors. To illustrate the advantages of incorporating the yield behavior of PL into models, the  
7 proposed PL model was compared with a basic PL acceleration-deceleration model only under  
8 the influence of PLL, and the results shows introducing the influence of the merging vehicle on  
9 its PL could more mimic the driving behavior of PL.

10 Nonetheless, the heterogeneity of the drivers is not incorporated in the present study,  
11 which is due to the lack of drivers' information in the NGSIM data. Further experimental data  
12 collection is worth executed to improve the models and increase prediction accuracy in the  
13 future. This paper only focuses on modeling the driving behavior of vehicles for the accepted  
14 gap. In future, we will try to build a more comprehensive lane change model including the  
15 rejected gap by parameterizing the gap selection.

## 16 REFERENCE

- 17 [1] Sun, D. and A. Kondyli. *Modeling Vehicle Interactions During Lane-Changing Behavior on*  
18 *Arterial Streets. Computer-Aided Civil and Infrastructure Engineering*, Vol. 25, No. 8, 2010,  
19 pp. 557-571.
- 20 [2] Daamen, W., M. Loot, and S.P. Hoogendoorn, *Empirical Analysis of Merging Behavior at*  
21 *Freeway On-Ramp*. Transportation Research Record, 2010, No.2188, pp. 108-118.
- 22 [3] Moridpour, S., M. Sarvi, and G. Rose, Modeling the Lane-Changing Execution of  
23 Multiclass Vehicles Under Heavy Traffic Conditions. Transportation Research Record, 2010,  
24 No.2161, pp. 11-19.
- 25 [4] Yeo, H., et al., *Oversaturated Freeway Flow Algorithm for Use in Next Generation*  
26 *Simulation*. Transportation Research Record, 2008, No.2088, pp. 68-79.
- 27 [5] Schakel, W.J., V.L. Knoop, and B.v. Arem, *LMRS: An Integrated Lane Change Model with*  
28 *Relaxation and Synchronization*, in TRB 2012 Annual Meeting. 2012: Washionton. D. C.
- 29 [6] Toledo, T., H. Koutsopoulos and M. Ben-Akiva. Modeling Integrated Lane-Changing  
30 Behavior. *Transportation Research Record: Journal of the Transportation Research Board*,  
31 Vol. 1857, No. -1, 2003, pp. 30-38.
- 32 [7] Hidas, P. Modelling Vehicle Interactions in Microscopic Simulation of Merging and  
33 Weaving. *Transportation Research Part C: Emerging Technologies*, Vol. 13, No. 1, 2005,  
34 pp. 37-62.
- 35 [8] Wang, X., T. Miyagi and J. Ying. A Simulation Model for Traffic Behavior at Merging  
36 Sections in Highways. Proceedings of the Second International Conference on Innovative  
37 Computing, *Information and Control*, 2007, pp. 30-43.
- 38 [9] Wan, X., J. Jin, L. Zheng, Y. Cheng and B. Ran. Empirical Analysis of Speed  
39 Synchronization of Merge Vehicle from Entrance Ramp. *Transportation Research Record:*  
40 *Journal of the Transportation Research Board*, Washington, D.C., 2013. (under press)
- 41 [10] Gunay, B. Car Following Theory with Lateral Discomfort. *Transportation Research Part B:*  
42 *Methodological*, Vol. 41, No. 7, 2007, pp. 722-735.
- 43 [11] Jin, S., D.-H. Wang and X.-R. Yang. Non-Lane-Based Car-Following Model with Visual  
44 Angle Information. *Transportation Research Record: Journal of the Transportation*

- 1        *Research Board*, Vol. 2249, No. -1, 2011, pp. 7-14.
- 2 [12] Kou, C.-C. and R. Machemehl. Modeling Driver Behavior During Merge Maneuvers. Book,  
3        Published by Southwest Region University Transportation Center, Center for Transportation  
4        Research, University of Texas, 1997.
- 5 [13] Michaels, R. M. and J. FAZIO. Driver Behavior Model of Merging. *Transportation*  
6        *Research Record: Journal of the Transportation Research Board*, No. 1213, 1989, pp. 4-10.
- 7 [14] Ahmed, K., M. Ben-Akiva, H. Koutsopoulos and R. Mishalani. Models for Freeway Lane  
8        Changing and Gap Acceptance Behavior. *In Proceedings of 13th International Symposium*  
9        *on Transportation and Traffic Theory*, 1996,
- 10 [15] M., S., C. A. and K. M. Modeling of Freeway Ramp Merging Process Observed During  
11        Traffic Congestion. *Transportation and traffic theory*, Vol. 15, 2002, pp. 483-502.
- 12 [16] Bunker, J. and R. Troutbeck. Prediction of Minor Stream Delays at a Limited Priority  
13        Freeway Merge. *Transportation Research Part B: Methodological*, Vol. 37, No. 8, 2003, pp.  
14        719-735.
- 15 [17] Evans, J. L., L. Elefteriadou and N. Gautam. Probability of Breakdown at Freeway Merges  
16        Using Markov Chains. *Transportation Research Part B: Methodological*, Vol. 35, No. 3,  
17        2001, pp. 237-254.
- 18 [18] Kita, H., Tanimoto, K., and Fukuyama, K. A Game Theoretical Analysis of Merging-  
19        Giveaway Interaction: A Joint Estimation Model. *Transportation and traffic theory*, Vol. 15,  
20        2002, pp. 503-518.
- 21 [19] Liu, H., W. Xin, Z. Adam and J. Ban. A Game Theoretical Approach for Modelling Merging  
22        and Yielding Behaviour at Freeway on-Ramp Sections. *17th International Symposium on*  
23        *Transportation and Traffic Theory*, London, UK, 2007,
- 24 [20] Choudhury, C., V. Ramanujam and M. Ben-Akiva. Modeling Acceleration Decisions for  
25        Freeway Merges. *Transportation Research Record: Journal of the Transportation Research*  
26        *Board*, Vol. 2124, No. -1, 2009, pp. 45-57.
- 27 [21] Sarvi, M. and M. Kuwahara. Microsimulation of Freeway Ramp Merging Processes under  
28        Congested Traffic Conditions. *Intelligent Transportation Systems, IEEE Transactions*, Vol.  
29        8, No. 3, 2007, pp. 470-479.
- 30 [22] Sarvi, M., O. Ejtemai and A. Zavabeti. Modelling Freeway Weaving Manoeuvre.  
31        *Australasian Transport Research Forum (ATRF)*, 34th, 2011, Adelaide, South Australia,  
32        Australia, 2011,
- 33 [23] Yeo, H. Asymmetric Microscopic Driving Behavior Theory. PHD thesis, University of  
34        California Transportation Center, UC Berkeley 2008.
- 35 [24] Ferrari, P. The Effect of Driver Behaviour on Motorway Reliability. *Transportation*  
36        *Research Part B: Methodological*, Vol. 23, No. 2, 1989, pp. 139-150.